AP Statistics Binomial Random Variable Summary Practice

Key

In problems 1 and 2, indicate whether a binomial distribution is a reasonable probability model for the random variable X. Give your reasons in each case.

1. A manufacturer produces a large number of toasters. From past experience, the manufacturer knows that approximately 2% are defective. In a quality control procedure, we randomly select 20 toasters for testing. We want to determine the probability that no more than one of these toasters is defective.

   <u>Yes</u>

   B - 2 outcomes: defective, not defective
   I - toasters are independent
   N - 20 toasters
   S - probability of being defective is always .02

2. Draw a card from a standard deck of 52 playing cards, observe the card, and replace the card within the deck. Count the number of times you draw a card in this manner until you observe a jack.

   <u>No</u>   N - no fixed number of trials

A fair coin is flipped 20 times. $B(\overset{n}{20}, \overset{p}{.5})$

3. Determine the probability that the coin comes up tails exactly 15 times.

   $P(X=15) = \text{binompdf}(20, .5, 15) = .0148$

4. Find the probability that the coin comes up tails at least 15 times. (Include enough details so that it can be understood how you arrived at your answer.)

   $P(X \geq 15) = 1 - P(X \leq 14) = 1 - \text{binomcdf}(20, .5, 14)$
   $= .0207$

5. Find the mean and standard deviation for the random variable X in this coin flipping problem

   $\mu_X = \underset{n}{20}(\underset{p}{.5}) = 10$ $\sigma_X = \sqrt{20(.5)(.5)} = 2.24$ $n \cdot p \cdot (1-p)$

A headache remedy is said to be 80% effective in curing headaches caused by simple nervous tension. An investigator tests this remedy on 100 randomly selected patients suffering from nervous tension.

6. Define the random variable being measured. X = the number of patients who experience headache relief

   What kind of distribution does X have? $B(100, .8)$

7. Calculate the mean and standard deviation of X.

   $\mu_X = 100(.8) = 80$ $\sigma_X = \sqrt{100(.8)(.2)} = 4$

8. Determine the probability that exactly 80 subjects experience headache relief with this remedy.

$P(X=80) = \text{binompdf}(100, .8, 80) = .0993$

9. What is the probability that between 75 and 90 (inclusive) of the patients will obtain relief? Justify your method of solution.

binomcdf(100, .8, 90) − binomcdf(100, .8, 74)

$P(75 \le X \le 90) = P(X \le 90) - P(X \le 74) = .9660$

10. Amarillo Slim, a professional dart player, has an 80% chance of hitting the bullseye on a dartboard with any throw. Suppose that he throws 10 darts, one at a time, at the dartboard. $B(10, .8)$

(a) Find the probability that Slim hits the bullseye exactly six times.

$P(X=6) = \text{binompdf}(10, .8, 6) = .0880$

(b) Find the probability that he hits the bullseye at least four times.

$P(X \ge 4) = 1 - P(X \le 3)$
$= 1 - \text{binomcdf}(10, .8, 3) = .9991$

(c) Compute the mean and variance of the number of bullseyes in 10 throws.

$\mu_X = np = 10(.8) = 8$ $\quad \sigma_X = \sqrt{10(.8)(.2)} = 1.26$
$\sqrt{(n)p(1-p)}$

Multiple-Choice Section

11. In a large population of college students, 20% of the students have experienced feelings of math anxiety. If you take a random sample of 10 students from this population, the probability that exactly 2 students have experienced math anxiety is
(a) 0.3020
(b) 0.2634
(c) 0.2013
(d) 0.5
(e) 1
(f) None of the above

$B(10, .2)$
$P(X=2) = \text{binompdf}(10, .2, 2)$
$= .3019898$

12. In a certain large population, 40% of households have a total annual income of over $70,000. A simple random sample of 4 of these households is selected. What is the probability that 2 or more of the households in the survey have an annual income of over $70,000?
(a) 0.3456
(b) 0.4000
(c) 0.5000
(d) 0.5248
(e) The answer cannot be computed from the information given.

$B(4, .4)$
$P(X \ge 2) = 1 - P(X \le 1) = 1 - \text{binomcdf}(4, .4, 1)$
$.5248$

13. A dealer in the Sands Casino in Las Vegas selects 40 cards from a standard deck of 52 cards. Let Y be the number of red cards (hearts or diamonds) in the 40 cards selected. Which of the following best describes this setting:

(a) Y has a binomial distribution with $n = 40$ observations and probability of success $p = 0.5$.
(b) Y has a binomial distribution with $n=40$ observations and probability of success $p = 0.5$, provided the deck is shuffled well.
(c) Y has a binomial distribution with $n=40$ observations and probability of success $p = 0.5$, provided after selecting a card it is replaced in the deck and the deck is shuffled well before the next card is selected.
(d) Y has a normal distribution with mean $p = 0.5$.

14. The probability that a three-year-old battery still works is 0.8. A cassette recorder requires four working batteries to operate. The state of batteries can be regarded as independent, and four three-year-old batteries are selected for the cassette recorder. What is the probability that the cassette recorder operates?

(a) 0.9984
(b) 0.8000
(c) 0.5904
(d) 0.4096
(e) The answer cannot be computed from the information given.

$B(4, .8)$

$P(X = 4) = \text{binompdf}(4, .8, 4)$

$.4096$

15. It has been estimated that about 30% of frozen chickens contain enough salmonella bacteria to cause illness if improperly cooked. A consumer purchases 12 frozen chickens. What is the probability that the consumer will have more than 6 contaminated chickens?

(a) 0.961
(b) 0.118
(c) 0.882
(d) 0.039
(e) 0.079

$B(12, .3)$

$P(X > 6) = 1 - P(X \leq 6)$

$= 1 - \text{binomcdf}(12, .3, 6)$

$=$

$.0386$

KEY

Key

Geometric Practice Worksheet

Cranky Mower- to start her old mower, Rita has to pull a cord and hope for some luck. On any particular pull, the mower has a 20% chance of starting. $p = .2$

a. Find the probability that the mower doesn't start until the third pull. Show your work.

$(.8)^2(.2) = .128$

b. What is the probability that it takes her 10 tries to start the mower? Show your work.

$(.8)^9(.2) = .0268$

Chips – Suppose a computer chip manufacturer rejects 2% of the chips produced because they fail presale testing.

$p = .02$

a. What's the probability that the fifth chip you test is the first bad one you find?

$(.98)^4(.02) = .0184$ geompdf(.02,5)

b. What is the probability that you find a bad one within the first 10 you examine?

$P(X \leq 10) = .02 + .98(.02) + \ldots + (.98)^9(.02)$

$= \text{geomcdf}(.02, 10) = .1829$

Roulette – Marti decides to keep placing a \$1 bet on number 15 in consecutive spins of a roulette wheel until she wins. On any spin, there's a 1-in-38 chance that the ball will land in the 15 slot.

a. How many spins do you expect it to take until Marti wins? Justify your answer.

$1/\frac{1}{38} = 38$

b. Would you be surprised if Marti won in 3 or fewer spins? Compute an appropriate probability to support your answer.

1 spin $= \frac{1}{38}$ + 2 spins $= \frac{37}{38}\left(\frac{1}{38}\right)$ + 3 spins $= \left(\frac{37}{38}\right)^2\left(\frac{1}{38}\right)$

$P(X \leq 3) = \text{geomcdf}\left(\frac{1}{38}, 3\right) = .0769$

Using Benford's Law – According to Benford's law, the probability that the first digit of the amount of a randomly chosen invoice is an 8 or a 9 is 0.097. Suppose you examine randomly selected invoices from a vendor until you find one whose amount begins with an 8 or a 9.

a. How many invoices do you $\overset{\mu}{\text{expect}}$ to examine until you get one that begins with an 8 or 9? Justify your answer.

$\frac{1}{.097} = 10.31$

b. In fact, you don't get an amount starting with an 8 or 9 until the 40th invoice. Do you suspect that the invoice amounts are not genuine? Compute an appropriate probability to support your answer.

$(1-.097)^{39}(.097) = .0018$